Mathematics of Topological Data Analysis (TDA): A Gentle Introduction (Module I)

Motivation & Syllabus Outline

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Contents

1 Module I: Geometric Foundations and Homotopy

- Combinatorics, Subcomplexes, Closures and Filtrations
- Geometric Realization and Simplicial Maps
- Barycentric Subdivision and Filtrations from Data (Vietoris-Rips and Čech)
- Basic Definitions of Homotopy and Contractible Spaces
- Carriers, Fibers, Nerves and the Nerve Theorem
- Elementary Collapses

1.1 Module Description

This module introduces the foundational combinatorial structures of simplicial complexes and explores when two topological spaces are equivalent through continuous deformation. It covers how to map discrete structures to continuous spaces via geometric realization, introduces filtrations, and studies contractible spaces. It also introduces powerful tools like the Carrier Lemma, Quillen's Fiber Theorem, and the nerves of open covers.

1.2 Relevance to TDA

Unions of geometric balls around data points are remarkably inconvenient for algorithms; this module provides the exact mathematical machinery to replace continuous unions with computable filtrations of simplicial complexes built directly from metric data spaces. Furthermore, the Nerve Theorem guarantees that these TDA methods are topologically sound, proving that simplified combinatorial "nerves" preserve the exact homotopy type of the underlying continuous shape.

Warning: Please do not be misled by the word "gentle" in the title. This course is both abstract and rigorous. The term reflects the way the material is introduced, with applications in mind, particularly beyond pure mathematics. As a result, the series deliberately avoids getting too much into the language of category theory and homological algebra, which are more traditional frameworks for introducing algebraic topology to graduate mathematics students.

2 Prerequisites

This module assumes a strong background in topology, particularly metric topology, that is, the theory of metric spaces and their induced topologies. A solid understanding of real analysis is therefore also highly beneficial. QF Academy already offers the following courses that can help build or refresh this background:

- Real Analysis (Module 1): Lecture 2, 3 & 5
- Group Theory, Topology & Manifolds: Topology Module
- Abstract Maths 101 Bootcamp: Topology Module

3 How to Enrol

This module will be rolled out gradually to our most active members starting from the week of February 16, 2026. To enrol, you must be subscribed to any of our existing plans that give you access to the Focused Tracks. Progression through the series is intentional and structured.

Enrolment in subsequent modules will be conditional on successfully completing this module. This is a deliberate design choice aimed at discouraging abundance learning where courses are started but not completed and understanding remains shallow. Members are free to retake this module as many times as needed until they meet the completion requirements. There is no penalty for revisiting the material. The objective is not speed but depth. In 2026, our goal is to significantly increase completion rates and learning outcomes across the academy. This requires focus, sustained effort, and respect for the learning process. By enforcing completion while allowing multiple attempts, we aim to support members in building genuine mathematical fluency rather than simply moving on without a solid foundation.