



Mathematics of Topological Data Analysis (TDA): A Gentle Introduction (Module II)

Motivation & Syllabus Outline

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1 Module II: Homology and Algebraic Machinery

- Euler Characteristic, Orientations, and Boundaries
- Chain Complexes, Homology Groups, and Betti Numbers
- The Smith Decomposition
- Categories, Functors, and Chain Maps
- Functoriality and Chain Homotopy
- The Snake Lemma, Pairs, Relative Homology, and the Mayer-Vietoris Sequence

1.1 Module Description

Because homotopy equivalence is difficult to compute directly, this module introduces homology as a computable algebraic invariant. It covers how to set up chain complexes of vector spaces using boundary operators, formally defining homology as the vector space of cycles modulo boundaries. It then introduces category theory to formally study structure-preserving maps, establishing the functoriality of homology, and utilizes exact sequences (like Mayer-Vietoris) to compute the homology of a complex by decomposing it into simpler pieces.

1.2 Relevance to TDA

Homology allows a computer to translate geometry into linear algebra; by reducing boundary matrices using the Smith Normal Form, TDA algorithms can explicitly calculate Betti numbers to count robust topological features like connected components and holes. Additionally, functoriality is the theoretical engine behind persistent homology, allowing us to track how a specific topological feature maps from one scale of a data filtration to the next.

2 Prerequisites

This module assumes a strong background in topology, particularly metric topology, that is, the theory of metric spaces and their induced topologies. A solid understanding of real analysis is therefore also highly beneficial. QF Academy already offers the following courses that can help build or refresh this background:

- Real Analysis (Module 1): Lecture 2, 3 & 5
- Group Theory, Topology & Manifolds: Topology Module
- Abstract Maths 101 Bootcamp: Topology Module

3 How to Enrol

This module will be rolled out gradually to our most active members starting from the week of February 16, 2026. To enrol, you must be subscribed to any of our existing plans that give you access to the Focused Tracks. Progression through the series is intentional and structured. Enrolment in subsequent modules will be conditional on successfully completing this module. This is a deliberate design choice aimed at discouraging abundance learning where courses are started but not completed and understanding remains shallow. Members are free to retake this module as many times as needed until they meet the completion requirements. There is no

penalty for revisiting the material. The objective is not speed but depth. In 2026, our goal is to significantly increase completion rates and learning outcomes across the academy. This requires focus, sustained effort, and respect for the learning process. By enforcing completion while allowing multiple attempts, we aim to support members in building genuine mathematical fluency rather than simply moving on without a solid foundation.