Mathematics of Topological Data Analysis (TDA): A Gentle Introduction (Module III)

Motivation & Syllabus Outline

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February 16, 2026

Contents

1 Module III: Cohomology and Persistence

- Cochain Complexes and Cohomology
- The Cup Product and The Cap Product
- Poincaré Duality
- Persistent Homology and Barcodes
- Algorithm for Filtrations
- Interleaving Distance, Bottleneck Distance, and The Stability Theorem

1.1 Module Description

This module first develops cohomology by dualizing chain complexes, allowing cochains to be multiplied together via cup and cap products to form a graded algebra. It then shifts to persistence, covering the Structure Theorem, which proves that persistence modules can be uniquely decomposed into interval modules called barcodes. It details the matrix reduction algorithm used to compute these barcodes and introduces the Stability Theorem, linking the interleaving distance of modules to the bottleneck distance of barcodes.

1.2 Relevance to TDA

This module contains the absolute core of modern TDA. The persistent homology barcodes isolate the "lifespan" of geometric features in a dataset across varying scales. The Stability Theorem is vital for real-world applications because it provides geometric robustness: it guarantees that small amounts of noise in input data will only cause strictly bounded changes in the resulting barcodes. Meanwhile, cohomological operations capture geometric intersections, allowing algorithms to distinguish complex spaces that basic homology cannot.

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2 Prerequisites

This module assumes a strong background in topology, particularly metric topology, that is, the theory of metric spaces and their induced topologies. A solid understanding of real analysis is therefore also highly beneficial. QF Academy already offers the following courses that can help build or refresh this background:

- Real Analysis (Module 1): Lecture 2, 3 & 5
- Group Theory, Topology & Manifolds: Topology Module
- Abstract Maths 101 Bootcamp: Topology Module

3 How to Enrol

This module will be rolled out gradually to our most active members starting from the week of February 16, 2026. To enrol, you must be subscribed to any of our existing plans that give you access to the Focused Tracks. Progression through the series is intentional and structured. Enrolment in subsequent modules will be conditional on successfully completing this module. This is a deliberate design choice aimed at discouraging abundance learning where courses are started but not completed and understanding remains shallow. Members are free to retake this

module as many times as needed until they meet the completion requirements. There is no penalty for revisiting the material. The objective is not speed but depth. In 2026, our goal is to significantly increase completion rates and learning outcomes across the academy. This requires focus, sustained effort, and respect for the learning process. By enforcing completion while allowing multiple attempts, we aim to support members in building genuine mathematical fluency rather than simply moving on without a solid foundation.