Group Theory 101 Bootcamp

Syllabus

Course Motivation

These live bootcamps are designed to lower the entry barrier for our more advanced courses, such as Lie Groups with Applications. In response to community feedback, we are offering an opportunity to build foundational confidence and fluency in key mathematical concepts through guided, interactive sessions led by PhD-level instructors. Participants will gain the skills necessary to thrive in our higher-level programmes and apply their learning in both academic and industry contexts.

Students will indicate their preferred days and times through a registration survey. We will do our best to accommodate those outside the consensus, provided there are at least three students with the same preferred time.

Fundamentals of Group Theory

- Definition of a group and motivating examples (symmetries, number systems)
- Group axioms with geometric and algebraic interpretations
- Subgroups, cosets, and quotient groups
- Homomorphisms and Isomorphisms

Group Actions and Representations

- Group actions: definition, orbits, stabilizers
- Visualizing actions of D_{2n}, $O(n)$, and $SO(n)$ on Euclidean space
- Cayley's theorem and examples of permutation representations
- Linear representations of groups

Structure Theory and Matrix Groups

- Normal subgroups and simple groups
- Jordan–Hölder theorem and group decompositions
- Special classes of groups (Abelian, nilpotent, solvable)

Applications

- Groups in Cryptography
- Groups in Quantum Computing
- Groups in Machine Learning