QFA A CRASH COURSE ON ORDINARY DIFFERENTIAL EQUATIONS DRAFT SYLLABUS

1. First Order Differential Equations

- Definition of ordinary and partial differential equations, order of a differential equation, linear ODEs.
- High-level questions and techniques for solving differential equations.
- Solving separable differential equations.
- Example of first-order linear ODE solving with an initial value.

2. Existence and Uniqueness Theorems

- Theorem: first-order linear ODE solutions - existence and uniqueness.
- Theorem: second-order linear ODE solutions - existence and uniqueness.
- Theorem: generalised first-order linear ODE solutions – existence and uniqueness.
- Demonstration of theorems with examples.

3. Second-Order Linear Homogeneous Equations and the Wronskian

- Definition of standard form, homogeneous and nonhomogeneous second-order ODEs.
- Characteristic equations for ODEs.
- The principle of superposition and its proof.
- The Wronskian determinant of a second-order ODE.
- Abel's Theorem and its proof.
- A theorem on linear combinations of second-order ODEs.

4. Complex and Repeated Roots of the Characteristic Equation

- The complex exponential function.
- Euler's formula and its solutions.
- Interpreting complex roots of the characteristic equation.
- Interpreting repeated roots of the characteristic equation.

- The reduction of order method for linear homogeneous equations.

5. Nonhomogeneous Second-Order Linear Differential Equations

- Theorem: the nonhomogeneous superposition principle. Proof of theorem.
- General solutions to a class of nonhomogeneous ODEs.
- The undetermined coefficients method.
- The variation of parameters method.
- Examples: finding particular solutions to a variety of second-order ODEs.

6. Power Series Solutions

- Review of power series terminology (convergence, divergence, absolute convergence, radius of convergence.
- Summing, multiplying, dividing power series.
- Differentiability of power series on radius of convergence.
- Analytic functions definition via Tayor expansion.
- The ratio test.
- Ordinary points of second-order homogeneous ODEs.
- Theorem: general solutions from ordinary points.