



Stochastic Processes (Random Walks): A Rigorous Introduction (Module I)

Motivation & Syllabus Outline

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1 Module I: Measure-Theoretic Foundations

1.1 Scope

This module builds the measure-theoretic language needed for modern probability: σ -algebras, probability measures, extension ideas, and canonical measures on \mathbb{R} .

1.2 Learning outcomes

By the end of Module I, participants should be able to:

- Construct sample spaces and σ -algebras for concrete experiments, and define a probability measure on them.
- Prove standard measure identities (finite additivity consequences, monotonicity, subadditivity) and continuity from above/below.
- Work confidently with Borel sets on \mathbb{R} , Dirac measures, discrete measures, and density-based measures.
- Recognise the role of extension arguments (algebra \rightarrow σ -algebra) and why continuity matters.

1.3 Lectures

1. Lecture I: Introduction

Coverage: Course overview, probability as measure, notation and expectations for later modules.

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2. Lecture II: Probability Measures

Topics: sample spaces, events as σ -algebras, probability measure axioms, continuity properties, extension motivation.

3. Lecture III: Measures on \mathbb{R}

Topics: Borel σ -algebra, Dirac measures, point masses, discreteness vs atoms, densities and the standard normal.

1.4 Suggested checkpoints

- Be able to write down $(\Omega, \mathcal{F}, \mathbb{P})$ for a new experiment without guessing.
- Be able to justify “this defines a probability measure” in one page: axioms + closure + continuity/extension comments.

2 How to Enrol

To enrol, you must be subscribed to any of our existing plans that give you access to the Focused Tracks. Progression through the series is intentional and structured. Enrolment in subsequent

modules will be conditional on successfully completing this module. This is a deliberate design choice aimed at discouraging abundance learning where courses are started but not completed and understanding remains shallow. Members are free to retake this module as many times as needed until they meet the completion requirements. There is no penalty for revisiting the material. The objective is not speed but depth. In 2026, our goal is to significantly increase completion rates and learning outcomes across the academy. This requires focus, sustained effort, and respect for the learning process. By enforcing completion while allowing multiple attempts, we aim to support members in building genuine mathematical fluency rather than simply moving on without a solid foundation.